MATRIX METHOD FOR RELATING BASE CURRENT RATIOS TO FIELD RATIOS OF AM DIRECTIONAL STATIONS

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Abstract

It was shown that there exist matrices that relate base currents and base voltages to the field ratios of a directional array. As a moment method program is running, the field and base current for each tower is recorded. Matrices, that relate base currents to field ratios, can be extracted from this data or by inverting a matrix. This method is an exact solution and requires only one run of the moment method program. For this reason, it is superior to other iterative processes.

Introduction

Since the invention of the microcomputer there has been much software written that will analyze wire antennas. More recently, Moment Method programs have become readily available to Consultants. Programs such as NEC and miniNEC have become very popular. Also available, from the author, is the program MMA, which employs the techniques described in this paper.

A method of moment program is designed to calculate currents on wire antennas given information about geometry, loads and voltage sources. For a consultant who designs AM directional patterns, the program in itself is of little use. In general, what is known about the system, is the array geometry and field ratios. If the voltage sources were known, the moment method program could be run to give the currents on each element in the array. Once the currents are found, the field ratios can easily be calculated. The voltage sources that produce a particular pattern are generally not known. What is needed, is a method of going in the opposite direction. That is, given the field ratios, determine the base voltages and currents that produce the pattern.

One method might be to take a guess at the base voltages and run the program. The program yields the currents on each element and the field ratios can be calculated. These calculated field ratios are then compared to the desired field ratios and a new estimate is made for the base voltages. This process can be very time consuming because of the time required to run the moment program. This could take many iterations to get an accurate solution for the base voltages.

Another possibility is to load the base with a very large resistive load. In this way the base currents will be in direct proportion to the base voltages used in the analysis. For many arrays (usually when the towers are less than 90 degrees and of equal height) the base currents are very close to the field ratios. In this way, a good first estimate can be used. Although this method shows an improvement over the first method, there are still some interactions that must be done to come to the final solution.

In this paper a matrix method for relating the field ratios to the voltage sources will be presented. The matrix method is more direct, and requires only one run of the method of moment program.

What is desired is a matrix that can be used to relate the field ratios to either the base voltage or current ratios. Written in matrix form would be:

$$[S] \times [F] = [V] \text{ and } [H] \times [F] = [I]$$

[F], [V], and [I] are vertical matrices of dimension n, where n is the number of elements in the array.

$$[F] = \begin{matrix} F_1 \\ F_2 \\ F_3 \\ \cdot \\ \cdot \\ \cdot \\ F_n \end{matrix}$$

where F_i is the field ratio of the ith element.

$$[V] = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{matrix}$$

where V_i is the base voltage ratio of the ith element.

$$[I] = \begin{matrix} I_1 \\ I_2 \\ I_3 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{matrix}$$

where I_i is the base current ratio of the ith element.

[S] and [H] are square matrices of dimension n x n.

$$[S] = \begin{matrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ S_{31} & S_{32} & S_{33} & \cdots & S_{3n} \\ & & & & \cdot \\ & & & & \cdot \\ & & & & \cdot \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nn} \end{matrix}$$

$$[H] = \begin{matrix} H_{11} & H_{12} & H_{13} & \cdots & H_{1n} \\ H_{21} & H_{22} & H_{23} & \cdots & H_{2n} \\ H_{31} & H_{32} & H_{33} & \cdots & H_{3n} \end{matrix}$$

0018-9316/89/0600-0172$01.00

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$H_{n1}\ H_{n2}\ H_{n3} \cdots H_{nn}$

To find the S and H matrices, the inverse of the S matrix will be found first. During this process, the admittance matrix of the system (Y Matrix) will be found. The inverse of the S matrix will be referred to as the T matrix. ([F] = [T] x [V]; $[S] = [T]^{-1}$) Once the T and Y matrices are found, all other matrices can be found by matrix inversion or multiplication.

Y and Z Matrix

To find the Y matrix, the moment method program can be executed n times, where n is the number of towers. Each program run will contain only one voltage source. A voltage source of 1 volt is put at the base of each tower in turn. The other towers which do not contain the source are shorted.

The moment method program will calculate the currents on each element in the system. Assuming tower 1 is the element with the voltage source, the current in the base of tower 1 is Y_{11}. The current in the base of tower 2 is Y_{21}. The current in the base of tower 3 is Y_{31}. The current in the base of tower i is Y_{i1}.

When tower 2 contains the voltage source, the current in the base of tower 1 is Y_{12}. The current in the base of tower 2 is Y_{22}. The current in the base of tower 3 is Y_{32}. The current in the base of tower i is Y_{i2}.

This process can then be repeated until the entire Y matrix is filled. Once the Y matrix is complete the Z matrix can be obtained by inverting the Y matrix.

It may seem necessary to run the moment method program n times for an n element array. This is not the case. As the program is executed, a source is put on each segment of each element in turn. The currents are then stored in a large matrix. Therefore, only one run of the moment program is necessary if the currents can be extracted from this matrix.

T and S matrix

The T matrix can be found by finding the field radiated by each tower under the condition that only one tower at a time contains a source. This can be done in one of two ways. Many Moment programs will calculate the field coming from each tower. This routine can be run to get the field from each tower. A second way will be to sum the currents on each tower. The sum of the currents on a tower is proportional to the field radiated by the tower. The only precaution is to make sure each current is weighted by the segment length it represents. This is needed only when unequal segment lengths are used. When tower 1 contains the voltage source the field for tower 1 is equal to T_{11}. The field for tower 2 is equal to T_{21}. The field for tower 3 is equal to T_{31}. The field for tower i is equal to T_{i1}. Mathematically the following is done.

$$[F] = [T] \times [V]$$

$$F_1 = T_{11} \times V_1 + T_{12} \times V_2 \cdots$$
$$F_2 = T_{21} \times V_1 + T_{22} \times V_2 \cdots$$
$$F_3 = T_{31} \times V_1 + T_{32} \times V_2 \cdots$$

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$$F_n = T_{n1} \times V_1 + T_{n2} \times V_2 \cdots$$

Since, for this run, $V_1 = 1$ and $V_k = 0$ for $k>1$, the above two equations reduce to the following.

$$F_1 = T_{11}$$
$$F_2 = T_{21}$$
$$F_3 = T_{31}$$

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$$F_n = T_{n1}$$

This process is repeated for tower two by placing a 1 volt source at tower 2 and shorting the other towers. Under these conditions the field radiated from tower 1 is T_{12} and the field radiated from tower 2 is T_{22}. The field radiated from tower i is T_{i2}. This process is completed until the entire T matrix is completed. The S matrix is obtained by inverting the T matrix.

Again it may seem that it will take many runs of the moment program to fill this matrix, but the currents are available with only one run of the program. It may be necessary to determine where these data are stored in a particular program.

H Matrix

At this point the S matrix is found. The S matrix relates the field to the base voltage.

$$[S] \times [F] = [V]$$

If each side of this equation is multiplied by the Y matrix the following is obtained.

$$[Y] \times [S] \times [F] = [Y] \times [V]$$

The Y matrix times the V matrix is the I matrix. The Y matrix times the S matrix is the H matrix.

$$[H] \times [F] = [I]$$

where $[H] = [Y] \times [S]$ and $[I] = [Y] \times [V]$

Example

To better understand the process of computing these matrices and calculating the base current and voltage ratios, consider the following example.

Height (Degrees)	Spacing (Degrees)	Field	Phase (Degrees)
90	0	1.0	0
120	90	.8	-90

For this example, the base current ratio, base voltage ratio, drive point impedances, and power distribution need to be calculated.

Table I contains the currents on each tower, if a method of moment program is run with 1 volt on tower 1 and tower 2 shorted.

Table I

Height Above Ground (Degrees)	Current REAL (X 10-3 Amps)	IMAG
80	1.58407	-2.42378
70	2.89501	-4.31180
60	4.09589	-5.92852
50	5.16858	-7.24966
40	6.08942	-8.24346
30	6.83432	-8.87772
20	7.38241	-9.12215
10	7.71787	-8.94618
0	7.83080	-8.09837
110	.28673	.11128
100	.53801	.21390
90	.78199	.31871
80	1.01555	.42418
70	1.23448	.52777
60	1.43434	.62635
50	1.61098	.71659
40	1.76072	.79528
30	1.88048	.85958
20	1.96780	.90718
10	2.02091	.93642
0	2.03874	.94628

Y_{11} will be the base current at tower 1 which is .00783 - j .00810. Y_{21} will be the base current at tower 2 which is .00204 + j .00095.

The segments for both towers are of equal length except for the segment at the base of each tower. This segment is half the length of the other segments on the tower. To get a representation of the field coming from each tower, the currents can be summed. The sum of the currents in the first tower is .0457 - j .0592 A. This sum only includes half the current at the base of tower 1. The sum of the currents in the second tower is .0156 + j .0069 A. This sum includes only half the current for the base of tower 2. T_{11} is then equal to .0457 - j .0592 and T_{21} is equal to .0156 + j .0069.

Table II contains the currents on each tower if a method of moment program is run with 1 volt on tower 2 and tower 1 shorted.

Table II

Height Above Ground (Degrees)	Current REAL (X 10-3 Amps)	IMAG
80	.41357	.19534
70	.75529	.35579
60	1.06798	.50164
50	1.34703	.63086
40	1.58642	.74088
30	1.77996	.82919
20	1.92231	.89275
10	2.00942	.93307
0	2.03874	.94628
110	.20878	-0.98161
100	.39638	-1.75425
90	.58232	-2.42360
80	.76364	-2.97923
70	.93650	-3.40771
60	1.09667	-3.69582
50	1.24009	-3.83255
40	1.36301	-3.80968
30	1.46219	-3.62110
20	1.53499	-3.25971
10	1.57946	-2.71278
0	1.59442	-1.74005

Y_{22} will be the base current at tower 2 which is .00159 - j .00174. Y_{12} will be the base current at tower 1 which is .00204 + j .00095.

To get a representation of the field coming from each tower, the currents can be summed. The sum of the currents in the first tower is .0119 + j .0056 A. This sum only includes half the current at the base of tower 1. The sum of the currents in the second tower is .0120 - j .0334 A. These sums include only half the currents for the base of towers 1 and 2. T_{22} is then equal to .0120 - j .0334 and T_{12} is equal to .0119 + j .0056. Inverting the T matrix gives the S matrix as follows.

S_{11} 7.433 + j 9.855
S_{12} 3.891 - j 2.407
S_{21} 4.984 - j 3.212
S_{22} 8.511 + j 24.61

Inverting the Y matrix gives the following Z matrix.

Z_{11} 47.625 + j 60.079
Z_{12} 40.824 - j 60.635
Z_{21} 40.824 - j 60.635
Z_{22} 219.55 + j 292.77

Multiplying the Y matrix by the S matrix yields the following H matrix.

H11 .1512 + j .01515
H12 .005047 + j .007862
H21 .008186 + j .01333
H22 .06660 + j .02320

With these matrices the operating parameters can be calculated. By multiplying the H matrix by the field parameters, the base current ratios are as follows.

Tower	Magnitude	Phase (Degrees)
1	1.000	0.0
2	.305	-60.2

By multiplying the S matrix by the field parameters, the base voltage ratios are as follows.

Tower	Magnitude	Phase (Degrees)
1	1.000	0.0
2	3.059	-72.9

Using the base current ratios and the Z matrix, the following operating parameters can be calculated.

Tower	Impedance (Ohms)	Power Ratio
1	37.8 + j 40.1	1.00
2	458.6 + j 310.1	1.13

Conclusion

To increase the power of a moment method program in analyzing AM directional arrays, a procedure was demonstrated for finding matrices that relate base currents and voltages to the field ratios. This procedure is an exact solution. It requires no iteration to obtain a solution. Also, only one run of a moment method program is necessary. After the matrices are found that relate base currents and voltages to the field ratios, the operating parameters for the array can easily be calculated.